Regulation, Volatility and Efficiency in Power Markets

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Outline

1. Introduction
2. Dynamics
3. Efficiency–Non-Volatility Trade-off
4. Efficiency Analysis - Intermittent Supplier’s Effect
5. Ancillary Supplier - Double Price Mechanism
6. Simulations
   - Intermittent Supplier
   - Ancillary Supplier
7. Large Population Analysis
8. Mean Field Continuous Time Market System
Market deregulation started in the 1980s
- Chile, UK, California, Northeastern US

Reasons:
- Increase in:
  - efficiency (lower cost)
  - generator availability
  - investment
  - better predictability of prices
  - lower volatility

ISOs, RTOs, & market mechanisms

Success (?):
- Nordic Countries, Argentina, Texas, Australia, EU, Japan, Brazil...

Failure:
- Volatility, blackouts
- Enron: Market failure?
1 – Some Examples

- **Illinois** [1]
- **East US** [2]
- **Ontario** [1]
- **The Netherlands** [1]
- **New Zealand** [3]
- **West Texas** [4]

1 – Power Markets vs Traditional Markets

- Dynamism and Friction
- Not storable
- Constraints:
  - Equality constraint: Kirchhoff’s laws
  - Inequality constraint:
    - Thermal
    - Voltage
    - Stability
    - Transmission lines
- Startup, shutdown costs
- Outages
- Strategic behaviour by market participants
1 – Intermittent Supply

- Intermittent supply (wind):
  - Cheap and clean once built
  - Unreliable: 15% cap. factor
  - Requires extra infrastructure
  - Happening big time in some places (Germany, West Texas, etc.)

- Nuclear power:
  - Carbon-clean alternative
  - Reliable: 93.8% cap. factor
  - Rising construction costs: 15% each year 2003-2007
  - Concerns esp. after Fukushima Daiichi
  - Out of fashion: Germany, US (cf. with UK’s Hinkley Point C).
1 Mathematical models

- We create mathematical models for markets
- A model is a model is a model
- Sandbox for trying ideas
- Data fitting for better models

Principles of (mathematical) modelling:

- Tractability: analytical or simulative
- Be able to say something interesting
- Balancing complexity and expressiveness
- Explain natural phenomena: Occam’s razor.

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1 – What are we trying to model?

*Generation cannot increase instantaneously.*

*Supply and demand are coupled by price.*

*The process is dynamic.*

*Our mathematical hammer: stochastic differential equations*

Question 1: What happens with more wind penetration?
Question 2: Can we have stable price and efficient markets?
Question 3: How much wind can we have?
Question 4: Should we have different price classes?
Question 5: Are we going to see price oscillations with many players?
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Dynamics:

- Process is defined by demand \( d_t \), supply \( s_t \), intermittent supply \( s^w_t \), and price \( p^e_t \).
- Dynamics of demand \( d_t \), supply \( s_t \), and intermittent supply \( s^w_t \) are subject to friction.

\[
\begin{align*}
\text{Demand change: } \quad &dd_t = f^d(d_t, p^e_t)dt + \sigma_ddw^d_t \\
\text{Supply change: } \quad &ds_t = f^s(s_t, p^e_t)dt + \sigma_sdw^s_t
\end{align*}
\]

Friction is modelled by having bounded derivative for \( f^d \) and \( f^s \).

\[
\text{Int. supply change: } \quad ds^w_t = \phi(s^w_t; \theta_w)dt + \sigma_wdw^w_t
\]

- \( f^d \) is a strictly decreasing function of \( p^e \)
- \( f^s \) is a strictly increasing function of \( p^e \)

A simple example is a linear function of the form
\[
f(x, p^e) = A(t)x + B(t)p^e \text{ with } A, B \text{ of class } C^1([0, T]).\]
2 – Dynamics

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An ideal market maker observes prices, demand, supply and sets the price as a control.

Price cannot change by too much:

\[ dp_t^e = u_t^e dt, \quad |u_t^e| \leq u_{max} \]

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2 – Loss Functions

- **Demand side loss:** (consumers)

\[ l^d = p^e \cdot (s + s^w) - v \cdot \min(d, s + s^w) + c_{bo}(r), \]

where \( r \triangleq s - d \) is the reserve.

- **Supplier’s loss:**

\[ l^s = c(s) - p^e \cdot s \]

- **Intermittent supplier’s loss:** (wind)

\[ l^w(s^w, p^e) = c_w(s^w) - p^e \cdot s^w \]

- \( c(\cdot) \): convex, strictly increasing on \( \mathbb{R}_+ \), \( c_w(\cdot) \) non-decreasing

\[ |c(s)| \leq |c|(1 + |s|^{k_1}), \quad |c_w(s)| \leq |c_w|(1 + |s|^{k_2}). \]

- **Blackout cost:** \( c_{bo}(r) \): convex, strictly decreasing on \( \mathbb{R}_- \),

\[ |c_{bo}(r)| \leq |c_{bo}|(1 + |r|^{k_3}), \text{ and zero on } \mathbb{R}_+. \]
Social cost:

\[ J = \mathbb{E} \int_0^\infty e^{-\delta t} \left[ -v \cdot \min(d_t, s_t + s^w_t) + c(s_t) + c_w(s^w_t) + c_{bo}(r_t) \right] dt \]

This process is determined by \( x \triangleq [d, s, p^e]^\top \), wind process \( s^w \) and the control \( u^e \).

\( J = J(x, s^w, u) \).

Our objective: Find control law \( u \) such that minimizes

\[ V(x, s^w) = \inf_u J(x, s^w, u) \]

\( V \) is known as the value function.

This looks like a classical control problem ...
2 – Optimal Control

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\[ dx_t = \psi(x_t, u_t^e) dt + G dw_t \]

\[ \psi(x_t, u_t^e) = \begin{pmatrix} f^d(d_t, p_t^e) \\ f^s(s_t, p_t^e) \\ u_t^e \end{pmatrix}, \quad G = \begin{pmatrix} \sigma_d & 0 & 0 \\ 0 & \sigma_s & 0 \\ 0 & 0 & 0 \end{pmatrix} \]

Since wind is not controllable, we can omit it.

We need to solve the Hamilton-Jacobi-Belmann (HJB) equation:

\[
\delta V - \inf_{u \in \mathcal{U}} \left\{ l(x, u^e, s^w) + (\partial_x V)\top \psi(x, u^e) \right\} - (\partial_{s^w} V) \phi(s^w) - \frac{1}{2} \text{Tr} \left( \partial_{xx} V G G\top \right) - \frac{1}{2} \sigma_w^2 \partial_{s^w s^w} V = 0
\]

Result: The PDE has a solution and it is unique.
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The HJB equation is very complex (viscosity solutions + perturbation method)

Turns out that a solution is unique and the \textit{optimal control} is

\[ u^* = -\text{sgn} \left( \partial_p V^p \right) u_{max} \]

“bang-bang” control

Conclusion: A unique solution exists, but in general is very hard to obtain.
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Conclusion: A unique solution exists, but in general is very hard to obtain.
Let’s solve what we can! (Recall: $x \triangleq [d, s, p^e]^{\top}$.)

- **Quadratic cost:**

$$J(x_0, s^w_0, u^e) = \mathbb{E} \int_0^{\infty} e^{-\delta t} \left[ x_t^\top Q(s^w) x_t + 2x_t^\top D(s^w) + r_e|u^e_t|^2 \right] dt$$

$r_e$ is the **volatility coefficient**

Note: $Q$ depends on intermittent supply $s^w$ and is therefore a stochastic function.

- **Linear dynamics:**

$$dx_t = (Ax_t + Bu^e_t + c)dt + Gdw_t$$

This is not a terrible approximation.
Value function:

\[ V(x, s^w) = x^\top K(s^w)x + 2x^\top S(s^w) + q(s^w) \]

Partial Differential Equations:

\[
\begin{align*}
\delta K(s^w) - K(s^w)A - A^\top K(s^w) + K(s^w)Br_e^{-1}B^\top K(s^w) \\
- (\partial_{s^w} K(s^w))\phi(s^w) - (\partial_{s^w}^2 s^w K(s^w))\frac{1}{2}\sigma_w^2 - Q(s^w) &= 0 \\
\delta S(s^w) + K(s^w)Br_e^{-1}B^\top S(s^w) - K(s^w)c - A^\top S(s^w) \\
- (\partial_{s^w} S(s^w))\phi(s^w) - (\partial_{s^w}^2 s^w S(s^w))\frac{1}{2}\sigma_w^2 - D(s^w) &= 0 \\
\delta q(s^w) + S^\top(s^w)Br_e^{-1}B^\top S(s^w) - 2S^\top(s^w)c - \text{Tr}(K(s^w)GG^\top) \\
- (\partial_{s^w} q(s^w))\phi(s^w) - (\partial_{s^w}^2 s^w q(s^w))\frac{1}{2}\sigma_w^2 &= 0 
\end{align*}
\]

Optimal control:

\[ u^{e*}_t = -\frac{1}{r_e} B^\top [K(s^w_t)x_t + S(s^w_t)] \]
Efficiency: $\triangleq -E \int_0^\infty e^{-\delta t} \left[ x_t^\top Q(s^w)x_t + 2x_t^\top D(s^w) \right] dt$

Volatility: $\triangleq E \int_0^\infty e^{-\delta t} |u_t^e|^2 dt.$

**Theorem**

*Efficiency is a decreasing function of the volatility coefficient $r_e$.***

$r_e = 0.01$

$r_e = 1000$
Theorem

There is an inherent trade-off between volatility and efficiency.

- For high efficiency, volatility should not be penalized \((r_e = 0)\).
- Fixed price policy is very inefficient!
4 – Efficiency Analysis

- **Intermittent process**: (Ornstein-Uhlenbeck)
\[ ds_t^w = -\rho^w(s_t^w - \theta^w)dt + \sigma^w dw_t^w \]

- **Hypothesis**: Marginal wind cost is cheaper than other sources
\[ c_w'(s) < c'(s) \text{ for all } s \geq 0 \]

- **Efficiencies**:
  - without Intermittent supplier: \( E(x; r_e) \)
  - with Intermittent supplier: \( E^w(x, s^w; r_e, \theta^w) \)

**Theorem**

The exists \( \zeta(r_e; \theta^w) \) such that if \( \sigma^w > \zeta(r_e; \theta^w) \). Then, for all initial states and parameters and all \( r_e > 0 \)

\[ E(x; r_e) > E^w(x, s^w; r_e, \theta^w). \]

**Conclusion**: Adding cheap highly volatile wind generator \( \rightarrow \) reduced efficiency
4 – Efficiency Analysis

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**Conclusion**: Adding cheap highly volatile wind generator \( \rightarrow \) reduced efficiency
Let’s add fast producer to compensate!

- **Ancillary dynamics:**
  \[ ds_t^a = u_t^a dt, \quad t \geq 0 \]
  (Controllable and deterministic; cf. wind.)

- **Price dynamics:**
  \[ dp_t^e = u_t^e dt, \quad t \geq 0 \]
  \[ p_t^a = c_a'(s_t^a), \quad t \geq 0 \text{ marginal cost} \]

- **Ancillary supplier’s loss**
  \[ l^a() = c_a(s^a) - p^a \cdot s^a \]
5 – Ancillary Supplier

■ Social cost:

\[
J = \mathbb{E} \int_0^\infty e^{-\delta t} \left[ -v \cdot \min(d_t, s_t + s^w_t + s^a_t) + c(s_t) + c_w(s^w_t) + c_a(s^a_t) \\
+ c_{bo}(r_t) + u_t^\top Ru_t \right] dt,
\]

where \( r_t = s_t + s^w_t + s^a_t - d_t \).

■ Control:

\[
R \triangleq \begin{bmatrix} r_e & 0 \\ 0 & r_a \end{bmatrix}
\]

\[
u \triangleq [u^e, u^a]^\top
\]
standard form:
\[
\psi = \begin{pmatrix} f^d(d, p^e) \\ f^s(s, p^e) \\ s^a \\ p^e \end{pmatrix}, \quad G = \begin{pmatrix} \sigma_d & 0 & 0 & 0 \\ 0 & \sigma_s & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}
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perturbed HJB equation:
\[
\delta V - \inf_{u \in U} \left\{ (\partial_{s^a} V^p) u^a + (\partial_{p^e} V^p) u^e + l(d, s, s^a, p^e, s^w) \right\} \\
- (\partial_d V^p) f^d(d, p^e) - (\partial_s V^p) f^s(s, p^e) - (\partial_{s^w} V^p) \phi(s^w) \\
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- \frac{1}{2} \epsilon^2 \partial_{p^e p^e} V^p = 0
\]

optimal control:
\[
u^* = \arg \inf_{u \in U} \left( u^\top R u + \begin{bmatrix} \partial_{p^e} V^p \\ \partial_{s^a} V^p \end{bmatrix}^\top u \right) = -\frac{1}{2} R^{-1} \begin{bmatrix} \partial_{p^e} V^p \\ \partial_{s^a} V^p \end{bmatrix}
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5 – Efficiency Analysis

- **Hypothesis:**
  \[ c'(s) < c_a'(s) \text{ for all } s \geq 0 \]

- **Efficiencies:**
  - **Without** ancillary supplier: \( E(x, s^w; r_e) \)
  - **With** ancillary supplier: \( E^a(x, s^a, s^w; R) \)

**Theorem**

Let \( s^a(0) = 0 \). Then, for all initial \( x \)

\[ E^a(x, s^a, s^w; r_e) > E(x, s^w; r_e) \]

for all \( r_e > 0 \).

Another result: If price volatility allows to roam free \((r_a \to 0)\) then the efficiency with and without ancillary is the same. But cost of ancillary is higher.
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Another result: If price volatility allows to roam free \( (r_a \to 0) \) then the efficiency with and without ancillary is the same. But cost of ancillary is higher.
Adding intermittent supplier → cheaper power but increased volatility.

With high enough stochasticity performance drops

This inefficiency can be offset with fast, expensive, and deterministic supply.

But how much ancillary?
5 – (Interim) Conclusion

- Adding intermittent supplier → cheaper power but increased volatility.
- With high enough stochasticity performance drops
- This inefficiency can be offset with fast, expensive, and deterministic supply.

- But how much ancillary?
\[ d_{k+1} = d_k - \rho (d_k - (\beta - p_k^e)) \Delta t + \sigma w_k^d \sqrt{\Delta t} \]
\[ s_{k+1} = s_k - \rho (s_k - (\gamma + p_k^e)) \Delta t + \sigma w_k^s \sqrt{\Delta t} \]
\[ s_{w,k+1} = s_{w,k} - \rho^w (s_{w,k} - \theta_w) \Delta t + \sigma w_{w,k} \sqrt{\Delta t} \]
\[ p_{k+1} = p_k + u_k \Delta t \]

\[ \rho = 0.02, \; \rho^w = 0.1, \; \beta = 250, \; \gamma = -70, \; \theta_w = 20, \; \Delta t = 0.001s, \]
\[ \sigma = 4, \; \sigma_w = 12, \; t_{final} = 500s, \; r_e = 10. \]
6 – Simulations

Market with 5% int. penetration

Market with 30% int. penetration
6 – Simulations

\[ r_e = 0.01 \]

\[ r_e = 100 \]
6 – Simulations

Single Pricing \( r_e = 1 \)
Trade-off Comparison
6 – Some answers

Question 1: What happens with more wind penetration?
First it gets better, then it gets worse

Question 2: Can we have stable prices and efficient markets?
No!

Question 3: How much wind can we have?
With more than 15%-20% we are hitting the volatility cliff.
At the volatility cliff every new wind MW installed requires a MW of oil/gas installed.

Question 4: Should we have different price classes?
Yes!
6 – Some answers

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Question 2: Can we have stable prices and efficient markets? No!

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Question 4: Should we have different price classes? Yes!
Question 1: What happens with more wind penetration? 
First it gets better, then it gets worse

Question 2: Can we have stable prices and efficient markets? 
No!

Question 3: How much wind can we have? 
With more than 15%-20% we are hitting the volatility cliff. At the volatility cliff every new wind MW installed requires a MW of oil/gas installed.

Question 4: Should we have different price classes? 
Yes!
So far we only had a single “standard” producer, a single consumer, a wind producer and an ancillary producer.

What about when we have many heterogeneous players? Each with different dynamics, incentives, and price curves.

Question: Are prices going to oscillate? Explode? Chaotic behavior?

A game theoretic approach

- Agents are coupled via the price function
- Price function:
  - Stochastic
  - Depends on other agents’ control actions
  - Tracking all actions for large number of players impossible
- Information on other agents is available only via commonly announced knowledge (price: $p_t$).
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Set of agents: Many suppliers and consumers.

Each supplier: dynamic process for supply and price (price curve more precisely).

Each consumer: dynamic process for demand, supply it receives, and price (price curve).

Market price is cleared.

Instantaneous (common) price is determined.
7 – Dynamic Game

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- Each supplier: dynamic process for supply and price (price curve more precisely).
- Each consumer: dynamic process for demand, supply it receives, and price (price curve).
- Market price is cleared
- Instantaneous (common) price is determined
Dynamics (in equations)

- Consumers:

\[ \begin{align*}
    dd_t^i &= f^d(d_t^i, p_t, \phi^d_i(p_t; p_t^d)) dt + \sigma_d dw_t^d, \quad i \in \mathbb{N}^d \\
    ds_t^{d_i} &= f^s(s_t^{d_i}, u_t^{d_i}, f^m) dt, \quad i \in \mathbb{N}^d \\
    dp_t^{d_i} &= u_t^{d_i} dt, \quad i \in \mathbb{N}^d
\end{align*} \]

- Suppliers:

\[ \begin{align*}
    ds_t^i &= f^s(s_t^i, p_t, \phi^s_i(p_t; p_t^s)) dt + \sigma_s dw_t^s, \quad i \in \mathbb{N}^s \\
    dp_t^s &= u_t^s dt, \quad i \in \mathbb{N}^s
\end{align*} \]

- Price:

\[ p_t = f^m\left(\left(\phi^d_i(p_t, p_t^d)\right)_{i \in \mathbb{N}^d}, \left(\phi^s_i(p_t, p_t^s)\right)_{i \in \mathbb{N}^s}\right) \]
7 – Costs

- **Loss functions**

\[ l^d(\cdot) = p_t \cdot s^d_t - v \cdot d^i_t + c_{bo}(s^d_t - d^i_t) \]
\[ l^s(\cdot) = c(s^i_t) - p_t \cdot s^i_t \]

- **cost functions**

\[ J_d(x^{d_i}_{t_0}) = \mathbb{E} \int_{t_0}^{T} \left[ l^d(d^i_t, s^d_t, p_t) + r|u^d_t|^2 \right] dt, \quad i \in \mathbb{N}^d \]
\[ J_s(x^{s_i}_{t_0}) = \mathbb{E} \int_{t_0}^{T} \left[ l^s(s^i_t, p_t) + r|u^{s_i}_t|^2 \right] dt, \quad i \in \mathbb{N}^s \]
7 – Single Agent System

**Dynamics**

\[ dx_t = (\psi(x_t) + u_t)dt + Gdw_t \]

**Cost function**

\[ J(x_{t_0}, u) = \mathbb{E} \int_{t_0}^{T} \left[ l(x_t) + ru_t^2 \right] dt \]

**Value function**

\[-\partial_t V - \inf_{u \in \mathcal{U}} \left\{ (\partial_x V)^\top (\psi + u_t) + ru_t^2 \right\} - \frac{1}{2} \text{Tr}(\partial_{xx}^2 VGG^\top) - l = 0 \]

**Control**

\[ u^*(x, t) = -\frac{1}{2r} \partial_x V(x, t) \]

**State evolution**

\[ \partial_t \zeta + \sum_{i=1}^{n} \partial_{x_i} \left( \psi_i - \frac{1}{r} \partial_{x_i} V \right) \zeta - \sum_{i=1}^{n} \frac{1}{2} \sigma_i^2 \partial_{x_i x}^2 \zeta = 0 \]
7 – Value Function Perturbation

Perturbed HJB equations
 Consumers:

\[-\partial_t V_{d_i} - \inf_{u_{d_i} \in U_i} \{(\partial_{x_{d_i}} V_{d_i})^\top \psi_{d_i} + r|u_{d_i}^t|^2\} - \frac{1}{2} \sigma_d^2 \partial_{dd} V_{d_i}^d - \frac{1}{2} \delta_1^2 \partial_{sd, s_i} V_{d_i}^d - \frac{1}{2} \delta_2^2 \partial_{p, p_i} V_{d_i}^d - l^d = 0, \quad i \in N^d\]

Producers:

\[-\partial_t V_{s_i} - \inf_{u_{s_i} \in U_i} \{(\partial_{x_{s_i}} V_{s_i})^\top \psi_{s_i} + r|u_{s_i}^t|^2\} - \frac{1}{2} \sigma_s^2 \partial_{ss} V_{s_i}^s - \frac{1}{2} \delta_3^2 \partial_{p, p_i} V_{s_i}^s - l^s = 0, \quad i \in N^s\]

Conclusion: Can compute the single agent problem.
Control actions

\[ u_{d_i}^*(x, t) = -\frac{1}{2r} \partial_{p_{d_i}} V_p(x_{d_i}^*, t), \quad i \in \mathbb{N}^d \]

\[ u_{s_i}^*(x, t) = -\frac{1}{2r} \partial_{p_{s_i}} V_p(x_{s_i}^*, t), \quad i \in \mathbb{N}^s \]

Theorem

An equilibrium exists, is unique and is attained by best-response.

But computing the equilibrium is exponentially hard in the number of players → not going to happen!
Control Actions

Control actions

\[ u^d_i^*(x, t) = -\frac{1}{2r} \partial_{p^d_i} V_p(x^d_i, t), \quad i \in \mathbb{N}^d \]

\[ u^s_i^*(x, t) = -\frac{1}{2r} \partial_{p^s_i} V_p(x^s_i, t), \quad i \in \mathbb{N}^s \]

Theorem

*An equilibrium exists, is unique and is attained by best-response.*

But computing the equilibrium is exponentially hard in the number of players → not going to happen!
8 – Mean Field Formulation

- Assume there are many players.

- Each player only sees its own state and the global price.

- Each player assumes the population behaves according to an expected behaviour distribution.
8 – Nonlinear Mean Field Equation System on \([t_0, T]\)

\[\begin{align*}
[HJB] \quad & - \partial_t V_p^d - (\partial_{x d} V^d) \top \psi^d \star - \frac{1}{4r} (\partial_p V^d)^2 - \frac{1}{2} \sigma_d^2 \partial_{dd}^2 V^d \\
& - \frac{1}{2} \epsilon_1^2 \partial_{s d s d} V^d - \frac{1}{2} \epsilon_2^2 \partial_{p d p d} V^d - l^d = 0 \\
[HJB] \quad & - \partial_t V_p^s - (\partial_{x s} V^s) \top \psi^s \star - \frac{1}{4r} (\partial_p V^s)^2 - \frac{1}{2} \sigma_s^2 \partial_{ss}^2 V^s \\
& - \frac{1}{2} \epsilon_3^2 \partial_{p s p s} V^s - l^s = 0 \\
[FKP] \quad & \partial_t \zeta^d + \partial_d (f^d \zeta^d) + \partial_{s d} (f^{s d} \zeta^d) - \frac{1}{2r} \partial_p (\partial_p V^d \zeta^d) \\
& - \frac{1}{2} \sigma_d^2 \partial_{dd}^2 \zeta^d - \frac{1}{2} \epsilon_1^2 \partial_{s d s d} \zeta^d - \frac{1}{2} \epsilon_2^2 \partial_{p d p d} \zeta^d = 0 \\
[FKP] \quad & \partial_t \zeta^s + \partial_s (f^s \zeta^s) - \frac{1}{2r} \partial_p (\partial_p V^s \zeta^s) - \frac{1}{2} \sigma_s^2 \partial_{ss}^2 \zeta^s \\
& - \frac{1}{2} \epsilon_3^2 \partial_{p s p s} \zeta^s = 0 \\
[Price] \quad & p_t = \int_{\mathbb{R}^3} \int_{\mathbb{R}^2} f_m \left( x_t^d, x_t^s \right) \zeta^d(x^d; t) \zeta^s(x^s; t) dx^d dx^s
\end{align*}\]

Solve a deterministic problem
Under quadratic cost and linear dynamics:

\[-\frac{ds_{\theta d}}{dt} = (A_{\theta d}^\top - K_{\theta d} B_{\theta d} r^{-1} B_{\theta d}^\top) s_{\theta d} + K_{\theta d} h_{\theta d}(\bar{p}_t) + D_{\theta d}(\bar{p}_t),\]

\[d\bar{x}_{\theta d} = A_{\theta d} \bar{x}_{\theta d} + B_{\theta d} u_{\theta d}^* (t) + h_{\theta d}(\bar{p}_t),\]

\[-\frac{ds_{\theta s}}{dt} = (A_{\theta s}^\top - K_{\theta s} B_{\theta s} r^{-1} B_{\theta s}^\top) s_{\theta s} + K_{\theta s} h_{\theta s}(\bar{p}_t) + D_{\theta s}(\bar{p}_t),\]

\[d\bar{x}_{\theta s} = A_{\theta s} \bar{x}_{\theta s} + B_{\theta s} u_{\theta s}^* (t) + h_{\theta s}(\bar{p}_t),\]

\[\bar{p}_t = \int_{\Theta_d} \int_{\Theta_s} \frac{1}{2} \left( p_{\theta d}^d - p_{\theta s}^s \right) dF_d(\theta_d) dF_s(\theta_s),\]

\[u_{\theta d}^* = -r^{-1} B_{\theta d}^\top (K_{\theta d} \bar{x}_{\theta d} + s_{\theta d}),\]

\[u_{\theta s}^* = -r^{-1} B_{\theta s}^\top (K_{\theta s} \bar{x}_{\theta s} + s_{\theta s}).\]
8 – $\epsilon$-Nash Equilibrium

Given all players strategies $(u_1, \ldots)$, we have a Nash equilibrium if no player can benefit by deviating.

We have an $\epsilon$-Nash Equilibrium is no player can benefit more than $\epsilon$ by deviating.

$$J_i(u_i^0, u_{-i}^0) - \epsilon \leq \inf_{u_i \in U_i} J_i(u_i, u_{-i}^0) \leq J_i(u_i^0, u_{-i}^0).$$
Theorem

Under technical conditions the best response control actions together with the LQG Mean Field Equation System generate a set of controls $\mathcal{U}_{MF}^N \triangleq (u_i^0)_{i \in N}, 1 \leq N < \infty$, such that

(i) The LQG MF equations have a unique solution.

(ii) All agent systems $S(D_i), 1 \leq i \leq N^d; S(S_i), 1 \leq i \leq N^s$, are second order stable.

(iii) $\{\mathcal{U}_{MF}^N; 1 \leq N < \infty\}$ yields an $\epsilon$-Nash equilibrium for all $\epsilon > 0$, i.e., for all $\epsilon > 0$, there exists $N(\epsilon)$ such that for all $N \geq N(\epsilon)$

$$J_i^N(u_i^0, u_{-i}^0) - \epsilon \leq \inf_{u_i \in \mathcal{U}_g^N} J_i^N(u_i, u_{-i}) \leq J_i^N(u_i^0, u_{-i}^0).$$

Recipe: Solve the MF equation, bid using the MF solution. No gaming!
Theorem

Under technical conditions the best response control actions together with the LQG Mean Field Equation System generate a set of controls \( U_{MF}^N \triangleq (u_i^0)_{i \in N}, 1 \leq N < \infty \), such that

(i) The LQG MF equations have a unique solution.

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\[
J_i^N(u_i^0, u_{-i}^0) - \epsilon \leq \inf_{u_i \in U^N_g} J_i^N(u_i, u_{-i}^0) \leq J_i^N(u_i^0, u_{-i}^0).
\]

Recipe: Solve the MF equation, bid using the MF solution. No gaming!
8 – Simulations

**Dynamics**
100 suppliers and 100 consumers

**Consumer’s valuation:**
- $50/MWh - 0 to 8
- $100/MWh - 8 to 18
- $150/MWh - 18 to 22
- $100/MWh - 22 to 24

price convergence
8 – Finite vs MF Comparison

Supply and Demand in the Decentralized Setup

Supply and Demand in Perfect Information Exchange
What have we learned?

- Inherent volatility-efficiency tradeoff
- Intermittent supply (wind) has an efficiency cliff
- Having a dual price system: ancillary and “normal” is better
- Large population markets can be stable
- Information sharing is not critical

On the todo list:

- Locational marginal pricing
- Congestion pricing
- Data-driven analysis
- Extreme events analysis (shocks)